# Lumped- and Distributed-Element Equivalent Circuits for Some Symmetrical Multiport Signal-Separation Structures

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***Abstract*—This paper presents an extension to the synthesis of the symmetrical analysis methods for modeling passive $n$-port junctions. Two approaches are discussed: the electrical one, based on consideration of even- and odd-propagation modes, and the geometrical one, based on space symmetry operators. The convenience of using more complete models for both analysis and synthesis is manifest in the choice of the simplest topological circuits of some well-known signal-separation structures. An example of the synthesis of a lumped quadrature coupler is carried out via the geometrical approach and the results compared to those of a previously reported design performed via the electrical approach. Likewise, a distributed circuit for a ring-style five-port four-way equal-power divider simpler than others exhibiting the same power-division characteristics is presented.**

***Index Terms*— Branching synthesis, multiport, signal separation, symmetrical analysis methods.**

## I. Introduction

AN EASY and direct form to deduce the scattering-matrix $S$ of a symmetrical and reciprocal $n$-port junction if amplitude and phase of power split are specified is by considering the unitary property of the $S$-matrix and by assuming the junction to be well matched and perfectly directive. However, this approach lends itself neither to a relation between the characteristic admittances of the network nor to a functional form which relates the $S$-matrix entries and eigenvalues; hence, the search for the different possible equivalent circuits is not immediate.

A more complete modeling for symmetrical multiports can be obtained by either the electrical (for junctions with less than five ports) or geometrical symmetry approach.

The robustness of symmetrical analysis methods is due to the theory of eigenvalue equations in which they are supported.

There are two well-known procedures for the electrical symmetry approach. The first approach is one in which a circuit composed of two parallel coupled transmission lines is terminated in a matched impedance on each port, and thus, is considered as a pair of simple two-ports [1], which can be independently analyzed to obtain their individual responses whose sum gives the total responses of the composed circuit, i.e., the signal-separation characteristics of the whole structure.

Manuscript received February 14, 1996; revised May 19, 1997. This work was supported in part by COFETEL-SCT, SEP, and CONACYT, México.

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Publisher Item Identifier S 0018-9480(97)06056-0.

The second approach is one in which a closed structure is equally terminated in a matched impedance on each port and partitioned into two cascades of two-ports [2], which can also be independently analyzed through the transmission or $ABCD$ matrix to obtain (after summing, as previously mentioned) the response at each port.

Although the electrical symmetry approach does not use the eigenvalue concept directly it considers a consequence—the even- and odd-propagation modes—and takes advantage of the linearity property of device to use the superposition principle.

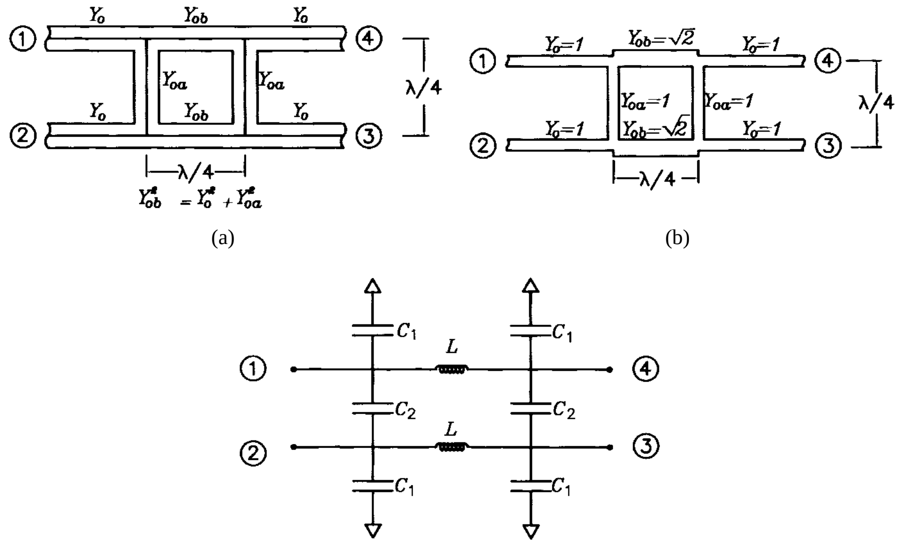
On the contrary, the geometrical symmetry approach is based on space symmetry operators and the groups they form [3], [4], and is linked to the electrical one through the commuting characteristic of the $S$-matrix with the symmetry operators. The $S$-matrix and the immittance $Y$- and $Z$-matrices are related via a matrix bilinear transformation [5], [6]. For a given symmetrical multiport, the operators of importance are classified as follows:

1) reflection in a plane;
2) rotation about an axis;
3) reflection in a point (which may be the origin of a coordinate system);
4) identity operator.

The application of any of these operators maintains the multiport unchanged or invariant (the port numeration is fixed in space) inasmuch as although the structure itself is transformed, and the current, voltage, and power at a port are replaced by those at another port, it is irrelevant due to the symmetry and reciprocity of the network.

A third often-used approach is based on *a priori* knowledge of the junction eigenadmittances [7], which gives rise to the notion of an equivalent admittance [8]. This approach focuses on a search for a two-port matching network which matches into this equivalent admittance. If this network is found, then the connection of one of the same kind in each port of a $n$-port junction allows the matching of the whole signal-separation structure, thus reducing a complicated multiport matched problem to a simpler one-port matching problem.

Nevertheless, all the methods have some drawbacks which limit their application. Thus, for instance in the geometrical case, if the junction under study does not have enough or *complete* symmetry, the eigenvalue formulation [3] cannot be used to predict its frequency dependence, and just as in the electrical case, the numerical values of $S$-matrix entries




(a) (b) (c)

Fig. 1. (a) Two-stub four-port directional coupler. (b) Distributed 3-dB directional quadrature coupler. (c) Lumped 3-dB directional quadrature coupler.

must be utilized instead to describe the theoretical frequency behavior of the network. On the other hand, in both the electrical case and the equivalent admittance case, the choice of the $S$-matrix that gives rise to the $Y$-matrix which represents the simplest junction from a topological point of view is not obvious. Besides, the electrical method can only be applied if the structure may be partitioned into two-ports.

Whatever the case, the enormous profit that has been obtained from the symmetry properties of some microwave networks to develop symmetrical analysis methods can be extended to the synthesis of several signal-separation structures.

## II. Synthesis via the Electrical Symmetry Approach

Fig. 1(a) shows a very popular structure: the two-stub four-port directional coupler. This junction has been synthesized as a distributed [2] or lumped [9] 3-dB directional quadrature coupler using the electrical symmetry approach.

The distributed circuit is constituted by four quarter-wavelength lines in a closed structure as shown in Fig. 1(b).

The lumped circuit is composed of an arrangement of series inductors and series and shunt capacitors as shown in Fig. 1(c).

However, neither the line arrangement of Fig. 1(b) nor the element arrangement of Fig. 1(c) are unique and, therefore, more possibilities of the same kind can be composed. These feasible networks are not evident but can be easily found by means of the generalized branching synthesis method presented in [10].

Thus, based on the information contained in [2] and considering the unitary property and a device well matched, the $S$-matrix form of a 3-dB directional quadrature coupler with equal-power distribution and 90° of relative phase between output ports may be derived as

$$S_{QC} = \begin{bmatrix} 0 & 0 & \delta & \epsilon \\ 0 & 0 & \epsilon & \delta \\ \delta & \epsilon & 0 & 0 \\ \epsilon & \delta & 0 & 0 \end{bmatrix}. \tag{1}$$

TABLE I
Value Combinations for the $\delta$ and $\epsilon$ Matrix Entries of a 3-dB Directional Quadrature Coupler (Double-Stub Four-Port Two-Way Equal-Power Divider)

| Comb. | $\lvert\delta\rvert$ = 0.707 | $\lvert\epsilon\rvert$ = 0.707 |
|---|---|---|
| - | arg[$\delta$] | arg[$\epsilon$] |
| 1 | 0° | 90° |
| 2 | 90° | 180° |
| 3 | 180° | 270° |
| 4 | 270° | 0° |
| 5 | 90° | 0° |
| 6 | 180° | 90° |
| 7 | 270° | 180° |
| 8 | 0° | 270° |

Then by using the simple searching algorithm of [10], eight different value combinations are found which can be assigned to $\delta$ and $\epsilon$, as shown in Table I. Here the equal-power distribution and the 90° of relative phase requires that $|\delta| = |\epsilon| = 0.707$ and $\arg[\delta] - \arg[\epsilon] = \pm 90°$. Also, because the 3-dB directional quadrature coupler is a two-stub four-port closed or ring structure (there exists a connection only between contiguous ports) and is assumed to be lossless, the $Y$-matrix must only be imaginary and have the following form:

$$Y_{QC} = j \begin{bmatrix} 0 & Y_{12} & 0 & Y_{14} \\ Y_{21} & 0 & Y_{23} & 0 \\ 0 & Y_{32} & 0 & Y_{34} \\ Y_{41} & 0 & Y_{43} & 0 \end{bmatrix}. \tag{2}$$

A different $Y$-matrix corresponds to each of these combinations and, hence, a different circuit in both lumped or distributed versions, as seen in Fig. 2. The choice of a particular circuit depends upon matters as the physical configuration of the network and its frequency behavior, which might be sound motives for choosing one in preference to the others.

Hence, in this simple searching algorithm, the quest is for sets of $S$-matrix entries which comply with the power and phase constraints and general admittance form of (2).

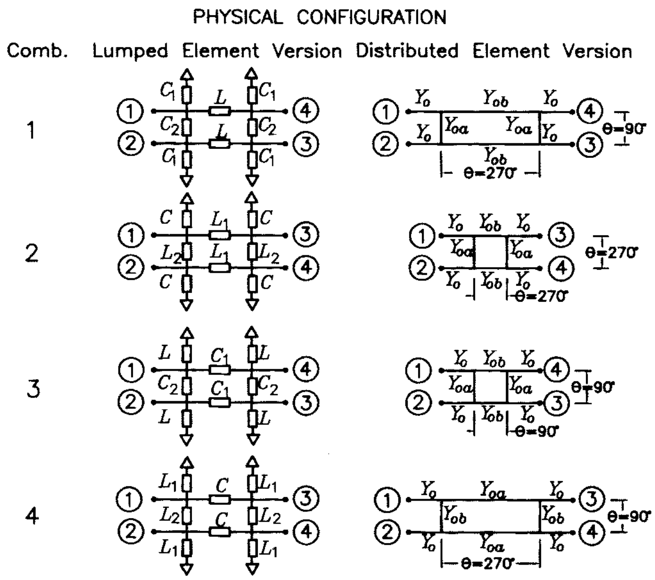



Fig. 2. Some feasible lumped and distributed networks for a 3-dB directional quadrature coupler.

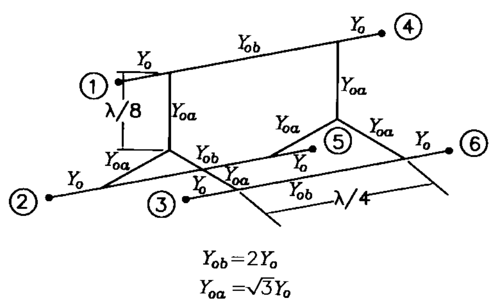



Fig. 3. Triple-line six-port directional coupler.

For distributed elements, combinations 1–4 result in the same networks as combinations 5–8, respectively, but some of the port numbers are changed. The $Y$ matrices corresponding to combinations 3 and 7 represent the better circuit configuration. All remaining combinations correspond to $Y$ matrices which represent networks with at least two long elements, which from a practical point of view are more frequency-sensitive.

For lumped elements, all combinations are possible, but as previously mentioned, a choice must be made considering physical feasibility and frequency dependence.

The variants of the distributed-element version may be directly inferred due to the transmission-line periodicity; however, those of the lumped-element version are not obvious at first sight.

Another well-known structure which has been synthesized as a stripline distributed directional coupler through the electrical symmetry approach, is the triple-line six-port directional coupler (Fig. 3) [11].

However, again the element arrangement of Fig. 3 is not unique and other feasible networks can be found.

The $S$-matrix form, as deduced to be applied in a three-conductor coaxial-line construction [12] of a triple-line six-port three-way equal-power divider, with a 120° phase difference between two of its output ports, is given by

$$S_{TL} = \begin{bmatrix} 0 & 0 & 0 & P & Q & Q \\ 0 & 0 & 0 & Q & P & Q \\ 0 & 0 & 0 & Q & Q & P \\ P & Q & Q & 0 & 0 & 0 \\ Q & P & Q & 0 & 0 & 0 \\ Q & Q & P & 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

TABLE II
VALUE COMBINATIONS FOR THE $P$ AND $Q$ MATRIX ENTRIES OF A TRIPLE-LINE SIX-PORT THREE-WAY EQUAL-POWER DIVIDER

| Comb. | $\|P\|$ = 0.577 | $\|Q\|$ = 0.577 |
|---|---|---|
| – | arg[$P$] | arg[$Q$] |
| 1 | 0° | 120° |
| 2 | 60° | 180° |
| 3 | 120° | 240° |
| 4 | 180° | 300° |
| 5 | 240° | 0° |
| 6 | 300° | 60° |
| 7 | 120° | 0° |
| 8 | 180° | 60° |
| 9 | 240° | 120° |
| 10 | 300° | 180° |
| 11 | 0° | 240° |
| 12 | 60° | 300° |

Then as before, by using the simple searching algorithm of [10], twelve possible value combinations are found for $P$ and $Q$, as shown in Table II.

For distributed elements, no combination may be practically realized in microstrip circuitry, but combination 10 provides a $Y$-matrix which represents a practical realizable stripline circuit (dual of network presented in [11]).

For lumped elements, combinations 1, 4, 8, and 11 lead to ring structures with a complicated (many crossed elements) inner star, combinations 2, 5, 7, and 10 lead to a network with two interconnected deltas, and combinations 3, 6, 9, and 12 lead to a ring structure with an inner star and a closeted or immerse port.

Fig. 4 shows the possible structures for both lumped and distributed elements.

## III. SYNTHESIS VIA THE GEOMETRICAL SYMMETRY APPROACH

The $S$ matrices and the functional relations between their entries and eigenvalues are given below for several well-known junctions.

Double-stub four-port directional coupler:

$$S_{DS} = \begin{bmatrix} \alpha & \beta & \delta & \epsilon \\ \beta & \alpha & \epsilon & \delta \\ \delta & \epsilon & \alpha & \beta \\ \epsilon & \delta & \beta & \alpha \end{bmatrix} \quad (4)$$

where

$$\alpha = \tfrac{1}{4}(s_1 + s_2 + s_3 + s_4) \quad (5)$$

$$\beta = \tfrac{1}{4}(s_1 - s_2 + s_3 - s_4) \quad (6)$$

$$\delta = \tfrac{1}{4}(s_1 + s_2 - s_3 - s_4) \quad (7)$$

$$\epsilon = \tfrac{1}{4}(s_1 - s_2 - s_3 + s_4). \quad (8)$$

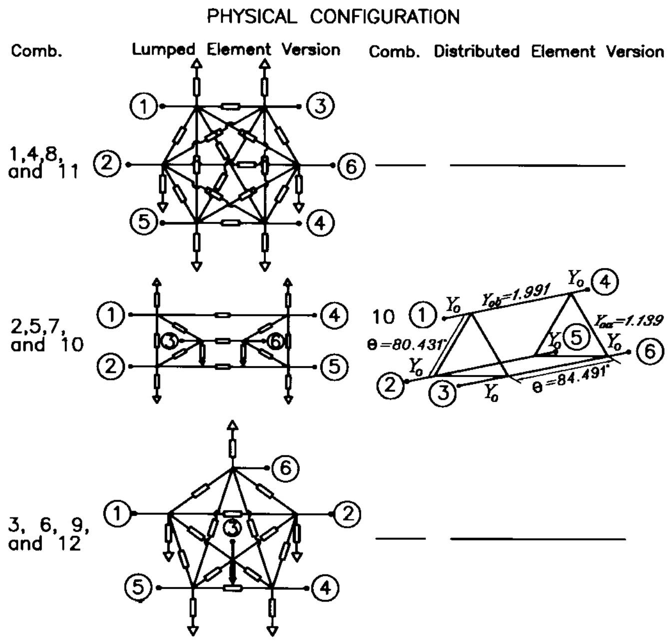



Fig. 4. Some possible lumped and distributed structures for a triple-line six-port directional coupler.

Ring-style five-port four-way equal-power divider:

$$S_{RF} = \begin{bmatrix} \sigma & \tau & \mu & \mu & \tau \\ \tau & \sigma & \tau & \mu & \mu \\ \mu & \tau & \sigma & \tau & \mu \\ \mu & \mu & \tau & \sigma & \tau \\ \tau & \mu & \mu & \tau & \sigma \end{bmatrix} \tag{9}$$

where

$$\sigma = \tfrac{1}{5}(s_1 + 2s_2 + 2s_3) \tag{10}$$
$$\tau = \tfrac{1}{5}[s_1 + (r_2 + r_5)s_2 + (r_3 + r_4)s_3] \tag{11}$$
$$\mu = \tfrac{1}{5}[s_1 + (r_3 + r_4)s_2 + (r_2 + r_5)s_3] \tag{12}$$

with

$$r_1 = 1 \quad r_2 = \exp(j\theta_1) \quad r_3 = \exp(j\theta_2)$$
$$r_4 = \exp(j\theta_3) \quad r_5 = \exp(j\theta_4)$$

and

$$\theta_1 = \tfrac{2}{5}\pi \quad \theta_2 = \tfrac{4}{5}\pi \quad \theta_3 = \tfrac{6}{5}\pi \quad \theta_4 = \tfrac{8}{5}\pi.$$

Ring-style six-port five-way equal-power divider:

$$S_{RS} = \begin{bmatrix} A & B & C & D & C & B \\ B & A & D & C & B & C \\ C & D & A & B & C & B \\ D & C & B & A & B & C \\ C & B & C & B & A & D \\ B & C & B & C & D & A \end{bmatrix} \tag{13}$$

where

$$A = \tfrac{1}{6}(s_1 + 2s_2 + 2s_3 + s_4) \tag{14}$$
$$B = \tfrac{1}{6}(s_1 + s_2 - s_3 - s_4) \tag{15}$$
$$C = \tfrac{1}{6}(s_1 - s_2 - s_3 + s_4) \tag{16}$$
$$D = \tfrac{1}{6}(s_1 - 2s_2 + 2s_3 - s_4). \tag{17}$$

TABLE III
VALUE COMBINATIONS FOR THE $\sigma = 0, \tau$, AND $\mu$ MATRIX ENTRIES OF A RING-STYLE FIVE-PORT FOUR-WAY EQUAL-POWER DIVIDER

| Comb. | $\lvert\tau\rvert$ = 0.5 | $\lvert\mu\rvert$ = 0.5 |
|---|---|---|
| – | arg[τ] | arg[μ] |
| 1 | 0° | 120° |
| 2 | 60° | 180° |
| 3 | 120° | 240° |
| 4 | 180° | 300° |
| 5 | 240° | 0° |
| 6 | 300° | 60° |
| 7 | 120° | 0° |
| 8 | 180° | 60° |
| 9 | 240° | 120° |
| 10 | 300° | 180° |
| 11 | 0° | 240° |
| 12 | 60° | 300° |

Purcell's junction (Special case: $E = G = 0$ reduces to a triple-line six-port directional coupler):

$$S_{PJ} = \begin{bmatrix} E & F & G & H & G & H \\ F & E & H & G & H & G \\ G & H & E & F & G & H \\ H & G & F & E & H & G \\ G & H & G & H & E & F \\ H & G & H & G & F & E \end{bmatrix} \tag{18}$$

where

$$E = \tfrac{1}{6}(s_1 + s_2 + 2s_3 + 2s_4) \tag{19}$$
$$F = \tfrac{1}{6}(s_1 - s_2 + 2s_3 - 2s_4) \tag{20}$$
$$G = \tfrac{1}{6}(s_1 + s_2 - s_3 - s_4) \tag{21}$$
$$H = \tfrac{1}{6}(s_1 - s_2 - s_3 + s_4). \tag{22}$$

The electrical symmetry approach can be applied to four-port multiple-stub couplers [2] and $n$-port multiple-line couplers [11], but not to five- and six-port ring structures because with a symmetrical partition it is not possible to obtain simple cascades of two-ports.

Besides, if in a similar way as for the 3-dB quadrature coupler, for junctions with more than four ports and through the electrical symmetry approach, it is attempted to directly search any circuit whatever its configuration. It is possible that this one have a complicated topology with crossed interconnections (a nonplanar network) and closeted or immerse ports.

Fortunately, by using geometrical space symmetry considerations, group theory, and the theory of the eigenvalue equations, the functional relations between $S$-matrix entries and eigenvalues are obtained, and thus, by means of the rigorous searching algorithm given in [10], it is easy to carry out an exhaustive search of all possible only reactive equivalent circuits.

The knowledge of these relations and of the $Y$-matrix configuration (composed of contiguous-port second-order interconnection submatrices), as well as the unitary condition (imaginary admittance matrix) of the $S$-matrix, allows one to establish this strict algorithm by applying shifts on the reference planes of each particular junction. The symmetric perturbations caused by these shifts correspond to rotations

TABLE IV
VALUE COMBINATIONS FOR THE $A = 0, B, C$, AND $D$ MATRIX ENTRIES OF A RING-STYLE SIX-PORT FIVE-WAY EQUAL-POWER DIVIDER

| Comb. | $\|B\|$ = 0.447 | $\|C\|$ = 0.447 | $\|D\|$ = 0.447 |
|---|---|---|---|
| - | arg[$B$] | arg[$C$] | arg[$D$] |
| 1 | 0° | 270° | 180° |
| 2 | 90° | 0° | 270° |
| 3 | 180° | 90° | 0° |
| 4 | 270° | 180° | 90° |
| 5 | 180° | 270° | 0° |
| 6 | 270° | 0° | 90° |
| 7 | 0° | 90° | 180° |
| 8 | 90° | 180° | 270° |

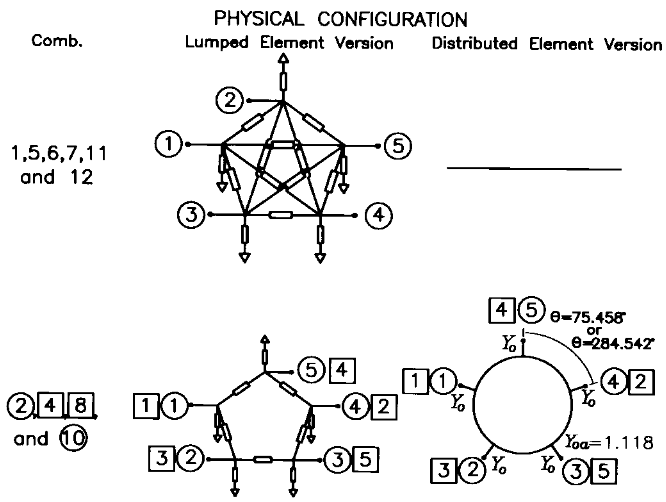



Fig. 5. Some possible lumped and distributed structures for a ring-style five-port four-way equal-power divider.

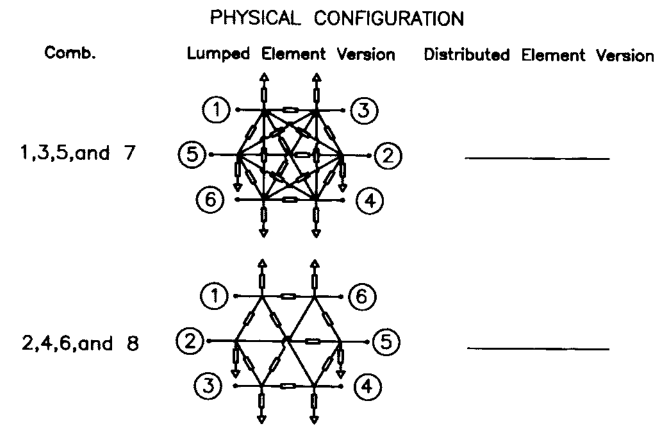



Fig. 6. Two possible lumped structures for a ring-style six-port five-way equal-power divider.

of the unit amplitude eigenvalues through arbitrary phase angles [7].

Therefore, in this rigorous searching algorithm, the quest is for eigenvalue phase angles which satisfy the $Y$-matrix configuration and the imaginary conditions.

Thus for instance, for the $S$-matrix of the ring-style five-port four-way equal-power divider (9), a possible set of entries derived to apply the equivalent admittance notion [13] is given by

$$\begin{aligned}\sigma &= 0\\ \tau &= -0.25 + j0.433 = 0.5\angle 120^\circ\\ \mu &= -0.25 - j0.433 = 0.5\angle 240^\circ.\end{aligned}$$

However, this set of numerical entries is one among several others, some of which are given for $\sigma = 0$ in Table III.

For distributed elements, combinations 4 and 10 correspond to $Y$-matrices giving the better circuit configurations, combinations 2 and 8 correspond to $Y$-matrices representing networks with more frequency-sensitive long elements, combinations 3 and 9 correspond to a quasi-singular $(I + S)$ matrix (where $I$ denotes the unity matrix), and all remaining combinations correspond to $Y$-matrices representing networks with long and/or wide–narrow elements and/or crossed interconnections, which will provide nonpractical realization in microstrip circuitry.

For lumped elements, all combinations (except 3 and 9) are possible, but again, a choice considering bandwidth and physical realizability must be made.

Fig. 5 shows the possible circuits for both lumped and distributed versions.

Some value combinations for the $S$-matrix entries of the ring-style six-port five-way equal-power divider (13) are shown for $A = 0$ in Table IV.

All combinations result in ring networks with crossed interconnections. For distributed elements, none is appropriate to be realized in microstrip circuitry. For lumped elements, the odd combinations of Table IV as those of Table III (except 3 and 9), give a ring structure with a inner star, as shown in Fig. 6. Even combinations give a ring structure with three internal crossed elements, as shown in Fig. 6.

Lastly, some value combinations for the $S$-matrix entries of the special case of Purcell's junction are shown for $E = G = 0$ in Table V.

All these combinations give the same networks as those of Table II and Fig. 4, but with a changed port numeration.

## IV. DIFFERENT ADMITTANCE MATRICES AND THE CIRCUITS THEY REPRESENT

Several equivalent circuits in both lumped and distributed versions have been shown in the last section for different signal-separation structures. The syntheses of these networks were carried out by means of the generalized branching synthesis method, in which for distributed elements the inter-

TABLE V
VALUE COMBINATIONS FOR THE $E = G = 0, F$, AND $H$ MATRIX ENTRIES OF A SPECIAL CASE OF PURCELL'S JUNCTION

| Comb. | \|F\| = 0.577 | \|H\| = 0.577 |
|---|---|---|
| – | arg[F] | arg[H] |
| 1 | 0° | 120° |
| 2 | 60° | 180° |
| 3 | 120° | 240° |
| 4 | 180° | 300° |
| 5 | 240° | 0° |
| 6 | 300° | 60° |
| 7 | 120° | 0° |
| 8 | 180° | 60° |
| 9 | 240° | 120° |
| 10 | 300° | 180° |
| 11 | 0° | 240° |
| 12 | 60° | 300° |

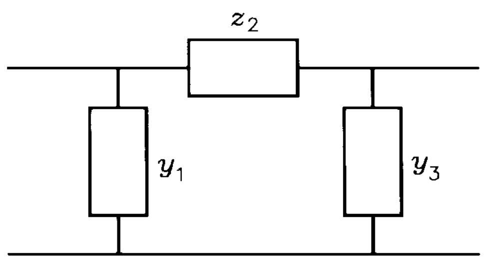



Fig. 7. Representation of a transmission-line section by a $\pi$ network.

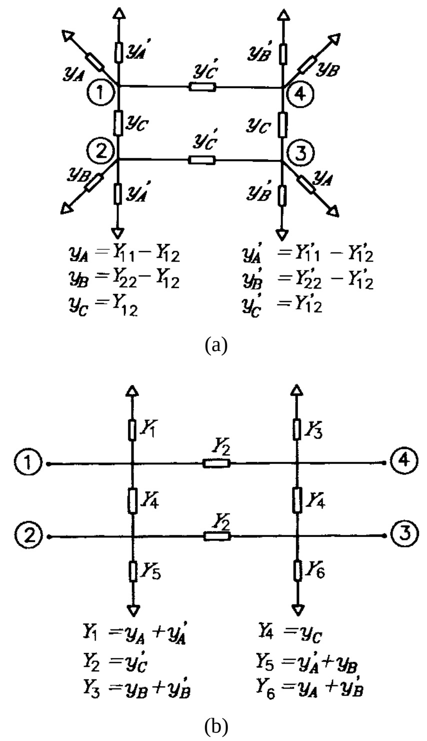



Fig. 8. Lumped circuit for a 3-dB directional quadrature coupler. (a) Initial circuit arrangement. (b) Reduced circuit with $Y_1 = Y_3 = Y_5 = Y_6 = j(\sqrt{2} - 1), Y_2 = -j\sqrt{2}$, and $Y_4 = j$.

connection admittance submatrices between ports are equated with the admittance matrix of a lossless transmission-line section to state a nonlinear system of equations from which the element physical values are obtained.

For lumped elements, the interconnection admittance submatrices are equated with the admittance matrix of a transmission-line section represented by a $\pi$ network (Fig. 7) [5], to obtain an arrangement of series and shunt inductors and capacitors.

Thus for instance, for the 3-dB directional quadrature coupler of Fig. 1(c), and for combination 1 of Table I, the $Y$-matrix as obtained by the bilinear transformation of the $S$-matrix of (1), is given by

$$Y_{QC} = j\begin{bmatrix} 0 & 1 & 0 & -\sqrt{2} \\ 1 & 0 & -\sqrt{2} & 0 \\ 0 & -\sqrt{2} & 0 & 1 \\ -\sqrt{2} & 0 & 1 & 0 \end{bmatrix}. \tag{23}$$

This matrix can be partitioned into four second-order interconnection submatrices [3] in the following form:

$$\begin{aligned} Y_{QC} = j\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + j\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 0 \\ 0 & -\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ + j\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} + j\begin{bmatrix} 0 & 0 & 0 & -\sqrt{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\sqrt{2} & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{24}$$

Equating the first and third with the $\pi$ network matrix expressed by

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \tag{25}$$

and the second and fourth with the $\pi$ network matrix expressed by

$$Y' = \begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{12} & Y'_{22} \end{bmatrix} \tag{26}$$

the following values are obtained:

$$\begin{gathered} Y_{11} = Y_{22} = Y'_{11} = Y'_{22} = 0 \\ Y_{12} = j1 \quad Y'_{12} = -j\sqrt{2}. \end{gathered}$$

Fig. 8 shows a circuit arrangement and its reduction considering these values.

Fig. 9 shows a final circuit for a design frequency of 60 MHz. The inductor and capacitor values are exactly the same as those given by Ho in [9].

Likewise, for the ring-style five-port four-way equal-power divider, and for combination 7 of Table III, the $Y$-matrix as obtained by the bilinear transformation of the $S$-matrix of (9), is given by

$$Y_{RF} = j\begin{bmatrix} a & b & c & c & b \\ b & a & b & c & c \\ c & b & a & b & c \\ c & c & b & a & b \\ b & c & c & b & a \end{bmatrix} \tag{27}$$

where $a = -5/\sqrt{3}$, $b = -2/\sqrt{3}$, and $c = 4/\sqrt{3}$.

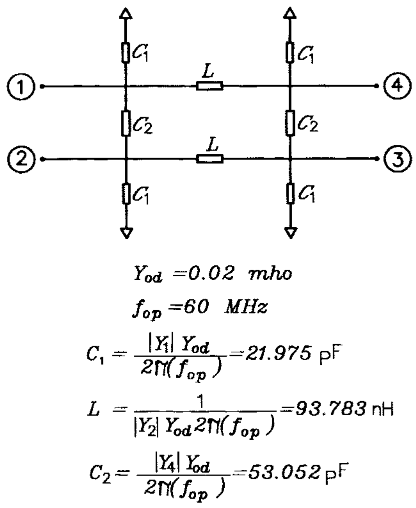



Fig. 9. Final circuit for a lumped 3-dB directional quadrature coupler operating at 60 MHz.

From this matrix ten submatrices of second order representing the interconnections between ports can be observed and are given in the following manner:

$$Y_{12} = Y_{15} = Y_{23} = Y_{34} = Y_{45} \tag{28}$$

$$Y_{13} = Y_{14} = Y_{24} = Y_{25} = Y_{35} \tag{29}$$

where

$$Y_{12} = j\begin{bmatrix} a/4 & b \\ b & a/4 \end{bmatrix} \tag{30}$$

and

$$Y_{13} = j\begin{bmatrix} a/4 & c \\ c & a/4 \end{bmatrix}. \tag{31}$$

Equating $Y_{12}$ with the matrix of (25), and $Y_{13}$ with the matrix of (26), the following values are obtained:

$$Y_{11} = Y_{22} = Y'_{11} = Y'_{22} = ja/4$$
$$Y_{12} = jb \quad Y'_{12} = -jc.$$

From these values a circuit arrangement and its reduction with an inner star (as shown in Fig. 10) are obtained.

A similar circuit with an inner star, but with noncrossed interconnections, was presented in [14].

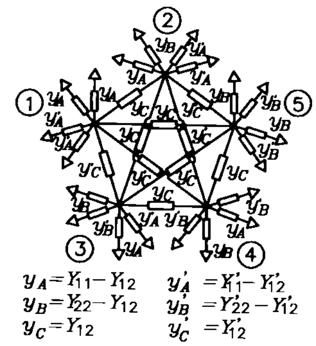
A circuit with the same power-division characteristics, but simpler from a topological point-of-view, can be obtained from combination 10 of Table III.

Thus, for this combination and from the $S$-matrix of (9), the corresponding $Y$-matrix can be given by

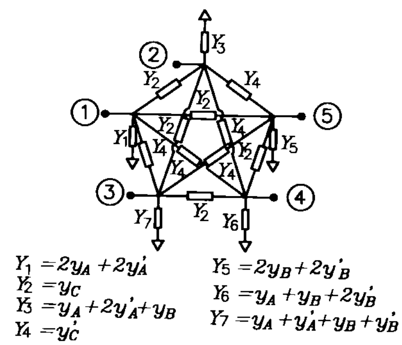
$$Y_{RFS} = j\begin{bmatrix} d & e & 0 & 0 & e \\ e & d & e & 0 & 0 \\ 0 & e & d & e & 0 \\ 0 & 0 & e & d & e \\ e & 0 & 0 & e & d \end{bmatrix} \tag{32}$$

where $d = -1/\sqrt{3}$ and $e = 2/\sqrt{3}$.

Effectuating a matrix partition as that of [10] and following the same procedure as above, a less complicated circuit arrangement and its reduction, as in Fig. 11, are obtained.

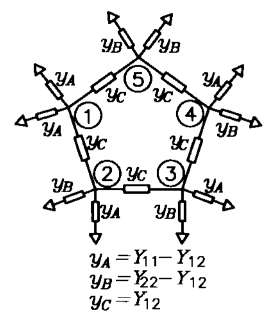



(a)

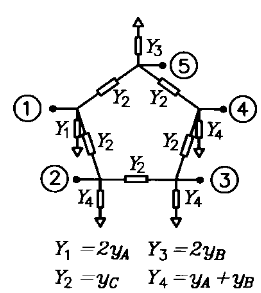



(b)

Fig. 10. Lumped circuit for a ring-style five-port four-way equal-power divider. (a) Initial circuit arrangement. (b) Reduced circuit with $Y_1 = Y_3 = Y_5 = Y_6 = Y_7 = j7/\sqrt{3}, Y_2 = -j2/\sqrt{3}$, and $Y_4 = -j4/\sqrt{3}$.




(a)




(b)

Fig. 11. Simpler lumped circuit for a ring-style five-port four-way equal-power divider. (a) Initial circuit arrangement. (b) Reduced circuit with $Y_1 = Y_3 = Y_4 = -j13/\sqrt{3}$ and $Y_2 = j6/\sqrt{3}$.

To the author's best knowledge, the physical configurations of the circuits of Figs. 4 and 5 (except the distributed-element version), 6, 10, and 11 are all new.

## V. Conclusion

A comparison among different forms to modeling symmetrical passive multiports has been presented. Models obtained with the geometrical symmetry approach have proven to be the powerful ones in applications to both the analysis and the synthesis techniques. Two very popular signal-separation structures, the two-stub four-port directional coupler and the ring-style five-port four-way equal-power divider (both in a lumped version) were synthesized in order to prove the statement. Thus, once the representative scattering matrix of a particular junction was obtained, the generalized branching synthesis method was applied to get different equivalent circuits among which the simplest one from a topological point-of-view was chosen.

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